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Question Paper Code : X67783

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Fourth Semester
Computer Science and Engineering
MA1252 – PROBABILITY AND QUEUEING THEORY
(Common to Information Technology)
(Regulations 2008)

Time : Three Hours Maximum : 100 Marks

Statistical Tables may be permitted.
Answer ALL questions

PART – A **(10×2=20 Marks)**

1. A random variable X takes the values 1, 2, 3, 4 such that 2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4). Find the probability distribution of X.

2. Write down the mean and variance of the Weibull distribution.

3. The joint probability mass function of a two dimensional random variable (X, Y) is given by p(x, y) = k(2x + 3y); x = 0, 1, 2; y = 1, 2, 3. Find the value of k.

4. What do you mean by correlation between two random variables ?

5. If N(t) is the Poisson process, then what can you say about the time we will wait for the first event to occur ? And the time we will wait for the n^{th} event to occur ?

6. Is Poisson process stationery ? Justify.

7. Define Markovian Queueing Models.

8. Suppose that customers arrive at a Poisson rate of one per every 12 minutes and that the service time is exponential at a rate of one service per 8 minutes.

9. State Pollaczek-Khinchine formula.

10. Define closed network of a queuing system.

PART – B **(5×16=80 Marks)**

11. a) i) If a random variable X has a cumulative distribution function $F(x)=\begin{cases}0, & \text{for } x \le 0\\ c\left(1-e^{x}\right) & \text{for } x>0\end{cases}$, find the probability density function, the value of c and P(1 < x < 2). **(8)**

ii) Find the moment generating function and r^{th} moment for the distribution whose probability density function is $f(x) = e^{-x}$, $0 \le x \le \infty$. Also find the first three moments about mean. **(8)**

(OR)

b) i) It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing at least, exactly and at most 2 defective items in a consignment of 1000 packets. **(8)**

ii) State and prove memory less property of exponential distribution. Using this property, solve the following problem :

The length of the shower on a tropical Island during rainy season has an exponential distribution with parameter 2, time being measured in minutes. If a shower has already lasted for 2 minutes, what is the probability that it will last for at least one more minute ? **(8)**

12. a) The joint probability density function of a two-dimensional random variable (X, Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8}; 0 \le x \le 2; 0 \le y \le 1$. Compute $P(X > 1)$, $P\left(Y < \frac{1}{2}\right), P\left(X > 1 / Y < \frac{1}{2}\right), P\left(Y < \frac{1}{2} / X > 1\right); P(X < Y)$ and $P(X + Y \le 1)$. **(16)**

(OR)

b) Obtain the equations of the regression lines from the following data. Hence find the coefficient of correlation between X and Y. Also estimate the value of Y when X = 38 and X when Y = 18. **(16)**

X :	22	26	29	30	31	31	34	35
Y :	20	20	21	29	27	24	27	31

13\. a) i) Prove that the process {X(t)} whose probability distribution given by

$$P\left[X(t)=n\right]=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,3,\ldots\\ \dfrac{at}{1+at}, & n=0\end{cases}$$ is not stationary. **(8)**

ii) The TPM of a Markov chain $\{X_n\}$, n = 1, 2, 3, ... having three states 1, 2 and 3 is $P=\begin{bmatrix}0.1 & 0.5 & 0.4\\0.6 & 0.2 & 0.2\\0.3 & 0.4 & 0.3\end{bmatrix}$ and the initial distribution is $P^{(0)} = \{0.7, 0.2, 0.1\}$ find

1) $P[X_2 = 3]$

2) $P[X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2]$. **(8)**

(OR)

b) i) A salesman territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However if he sells in either B or C the next day he is twice as likely as to sell in city A as in the other city. In the long run how often does he sell in each the cities ? **(8)**

ii) Suppose that customers arrive at a bank according to a Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes.

1) Exactly 4 customers arrive

2) More than 4 customers arrive

3) Less than 4 customers arrive. **(8)**

14\. a) Obtain the steady state probabilities of birth-death process. Also draw the transition graph. **(16)**

(OR)

b) At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20 kms down the river. Tankers arrive according to Poisson process with a mean of 1 every 2 hrs. It takes for an unloading crew, on the average, 10 hrs to unload a tanker, the unloading time following an exponential distribution. Find

i) how many tankers are at the port on the average ?

ii) how long does a tanker spend at the port on the average ?

iii) what is the average arrival rate at the overflow facility ? **(16)**

15. a) Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process at the rate of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The service time for all cars is constant and equal to 10 minutes. Determine L_s, L_q, W_s and W_q. **(16)**

(OR)

b) Consider a system of two servers where customers from outside the system arrive at sever 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates for server 1 and 2 are 8 and 10 respectively. A customer upon completion of service at server 1 is likely to go to server 2 or leave the system; whereas a departure from server 2 will go 25 percent of the time to server 1 and will depart the system otherwise. Determine the limiting probabilities, L_s and W_s. **(16)**

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